EXAMPLE 4.1: FRM EXAM 2009—QUESTION 14

Suppose you simulate the price path of stock HHF using a geometric Brownian motion model with drift $\mu = 0$, volatility $\sigma = 0.14$, and time step $\Delta t = 0.01$. Let S_t be the price of the stock at time t. If $S_0 = 100$, and the first two simulated (randomly selected) standard normal variables are $\epsilon_1 = 0.263$ and $\epsilon_2 = -0.475$, what is the simulated stock price after the second step?

a. 96.79
b. 99.79
c. 99.97
d. 99.70

EXAMPLE 4.2: FRM EXAM 2003—QUESTION 40

In the geometric Brown motion process for a variable S,

I. S is normally distributed.
II. $d\ln(S)$ is normally distributed.
III. dS/S is normally distributed.
IV. S is lognormally distributed.

a. I only
b. II, III, and IV
c. IV only
d. III and IV

EXAMPLE 4.3: FRM EXAM 2002—QUESTION 126

Consider that a stock price S that follows a geometric Brownian motion $dS = aSdt + bSdz$, with b strictly positive. Which of the following statements is *false*?

a. If the drift a is positive, the price one year from now will be above today's price.
b. The instantaneous rate of return on the stock follows a normal distribution.
c. The stock price S follows a lognormal distribution.
d. This model does not impose mean reversion.

EXAMPLE 4.4: INTEREST RATE MODEL

The Vasicek model defines a risk-neutral process for r that is $dr = a(b - r)dt + \sigma dz$, where a, b, and σ are constant, and r represents the rate of interest. From this equation we can conclude that the model is a

a. Monte Carlo type model
b. Single-factor term-structure model
c. Two-factor term-structure model
d. Decision tree model

EXAMPLE 4.5: INTEREST RATE MODEL INTERPRETATION

The term $a(b - r)$ in the previous question represents which term?

a. Gamma
b. Stochastic
c. Reversion
d. Vega

EXAMPLE 4.6: FRM EXAM 2000—QUESTION 118

Which group of term-structure models do the Ho-Lee, Hull-White, and Heath-Jarrow-Morton models belong to?

a. No-arbitrage models
b. Two-factor models
c. Lognormal models
d. Deterministic models

EXAMPLE 4.7: FRM EXAM 2000—QUESTION 119

A plausible stochastic process for the short-term rate is often considered to be one where the rate is pulled back to some long-run average level. Which one of the following term-structure models does *not* include this characteristic?

a. The Vasicek model
b. The Ho-Lee model
c. The Hull-White model
d. The Cox-Ingersoll-Ross model

EXAMPLE 4.8: FRM EXAM 2005—QUESTION 67

Which one of the following statements about Monte Carlo simulation is *false*?

a. Monte Carlo simulation can be used with a lognormal distribution.
b. Monte Carlo simulation can generate distributions for portfolios that contain only linear positions.
c. One drawback of Monte Carlo simulation is that it is computationally very intensive.
d. Assuming the underlying process is normal, the standard error resulting from Monte Carlo simulation is inversely related to the square root of the number of trials.

EXAMPLE 4.9: FRM EXAM 2007—QUESTION 66

A risk manager has been requested to provide some indication of the accuracy of a Monte Carlo simulation. Using 1,000 replications of a normally distributed variable S, the relative error in the one-day 99% VAR is 5%. Under these conditions,

- **a.** Using 1,000 replications of a long option position on S should create a larger relative error.
- **b.** Using 10,000 replications should create a larger relative error.
- **c.** Using another set of 1,000 replications will create an exact measure of 5.0% for relative error.
- **d.** Using 1,000 replications of a short option position on S should create a larger relative error.

EXAMPLE 4.10: SAMPLING VARIATION

The measurement error in VAR, due to sampling variation, should be greater with

- **a.** More observations and a high confidence level (e.g., 99%)
- **b.** Fewer observations and a high confidence level
- **c.** More observations and a low confidence level (e.g., 95%)
- **d.** Fewer observations and a low confidence level

EXAMPLE 4.11: FRM EXAM 2007—QUESTION 28

Let N be a $1 \times n$ vector of independent draws from a standard normal distribution, and let V be a covariance matrix of market time-series data. Then, if L is a diagonal matrix of the eigenvalues of V, E is a matrix of the eigenvectors of V, and $C'C$ is the Cholesky factorization of V, which of the following would generate a normally distributed random vector with mean zero and covariance matrix V to be used in a Monte Carlo simulation?

- **a.** $NC'CN'$
- **b.** NC'
- **c.** $E'LE$
- **d.** Cannot be determined from data given

EXAMPLE 4.12: FRM EXAM 2006—QUESTION 82

Consider a stock that pays no dividends, has a volatility of 25% pa, and has an expected return of 13% pa. The current stock price is $S_0 = \$30$. This implies the model $S_{t+1} = S_t(1 + 0.13\Delta t + 0.25\sqrt{\Delta t}\,\epsilon)$, where ϵ is a standard normal random variable. To implement this simulation, you generate a path of the stock price by starting at $t = 0$, generating a sample for ϵ, updating the stock price according to the model, incrementing t by 1, and repeating this process until the end of the horizon is reached. Which of the following strategies for generating a sample for ϵ will implement this simulation properly?

- **a.** Generate a sample for ϵ by using the inverse of the standard normal cumulative distribution of a sample value drawn from a uniform distribution between 0 and 1.
- **b.** Generate a sample for ϵ by sampling from a normal distribution with mean 0.13 and standard deviation 0.25.
- **c.** Generate a sample for ϵ by using the inverse of the standard normal cumulative distribution of a sample value drawn from a uniform distribution between 0 and 1. Use Cholesky decomposition to correlate this sample with the sample from the previous time interval.
- **d.** Generate a sample for ϵ by sampling from a normal distribution with mean 0.13 and standard deviation 0.25. Use Cholesky decomposition to correlate this sample with the sample from the previous time interval.

EXAMPLE 4.13: FRM EXAM 2006—QUESTION 83

Continuing with the previous question, you have implemented the simulation process discussed earlier using a time interval $\Delta t = 0.001$, and you are analyzing the following stock price path generated by your implementation.

t	S_{t-1}	ϵ	ΔS
0	30.00	0.0930	0.03
1	30.03	0.8493	0.21
2	30.23	0.9617	0.23
3	30.47	0.2460	0.06
4	30.53	0.4769	0.12
5	30.65	0.7141	0.18

Given this sample, which of the following simulation steps most likely contains an error?

- **a.** Calculation to update the stock price
- **b.** Generation of random sample value for ϵ
- **c.** Calculation of the change in stock price during each period
- **d.** None of the above

4.5 ANSWERS TO CHAPTER EXAMPLES

Example 4.1: FRM Exam 2009—Question 14

d. The process for the stock prices has mean of zero and volatility of $\sigma\sqrt{\Delta t} = 0.14\sqrt{0.01} = 0.014$. Hence the first step is $S_1 = S_0(1 + 0.014 \times 0.263) = 100.37$. The second step is $S_2 = S_1(1 + 0.014 \times -0.475) = 99.70$.

Example 4.2: FRM Exam 2003—Question 40

b. Both dS/S or $d\ln(S)$ are normally distributed. As a result, S is lognormally distributed. The only incorrect answer is I.

Example 4.3: FRM Exam 2002—Question 126

a. All the statements are correct except a., which is too strong. The expected price is higher than today's price but certainly not the price in all states of the world.

Example 4.4: Interest Rate Model

b. This model postulates only one source of risk in the fixed-income market. This is a single-factor term-structure model.

Example 4.5: Interest Rate Model Interpretation

c. This represents the expected return with mean reversion.

Example 4.6: FRM Exam 2000—Question 118

a. These are no-arbitrage models of the term structure, implemented as either one-factor or two-factor models.

Example 4.7: FRM Exam 2000—Question 119

b. Both the Vasicek and CIR models are one-factor equilibrium models with mean reversion. The Hull-White model is a no-arbitrage model with mean reversion. The Ho-Lee model is an early no-arbitrage model without mean reversion.

Example 4.8: FRM Exam 2005—Question 67

b. MC simulations do account for options. The first step is to simulate the process of the risk factor. The second step prices the option, which properly accounts for nonlinearity.